different observer. I cannot help entertaining the hope that something of the sort will sooner or later be undertaken with regard to the investigation of the whole nervous system."

May 19, 1870.

General Sir EDWARD SABINE, K.C.B., President, in the Chair.

The following communications were read :—

I. "A Ninth Memoir on Quantics." By Prof. CAYLEY, F.R.S. Received April 7, 1870.

(Abstract.)

It was shown not long ago by Prof. Gordan that the number of the irreducible covariants of a binary quantic of any order is finite (see his memoir "Beweis das jede Covariante und Invariante einer binären Form eine ganze Function mit numerischen Coefficienten einer endlichen Anzahl solcher Formen ist," Crelle, t. 69 (1869), Memoir dated 8 June 1868), and in particular that for a binary quantic the number of irreducible covariants (including the quantic and the invariants) is = 23, and that for a binary sextic the number is = 26. From the theory given in my "Second Memoir on Quantics," Phil. Trans. 1856, I derived the conclusion, which as it now appears was erroneous, that for a binary quintic the number of irreducible covariants was infinite. The theory requires, in fact, a modification, by reason that certain linear relations, which I had assumed to be independent, are really not independent, but, on the contrary, linearly connected together: the interconnexion in question does not occur in regard to the quadric, cubic, or quartic; and for these cases respectively the theory is true as it stands; for the quintic the interconnexion first presents itself in regard to the degree 8 in the coefficients, and order 14 in the variables; viz. the theory gives correctly the number of covariants of any degree not exceeding 7, and also those of the degree 8, and order less than 14; but for the order 14 the theory as it stands gives a non-existent irreducible covariant $(a, \ldots)^8(x, y)^{14}$; viz. we have, according to the theory, $5=(10-6)+1$, that is, of the form in question there are 10 composite covariants connected by 6 syzygies, and therefore equivalent to $10-6, =4$ asyzygetic covariants; but the number of asyzygetic covariants being $= 5$, there is left, according to the theory, 1 irreducible covariant of the form in question. The fact is that the 6 syzygies being interconnected and equivalent to 5 independent syzygies only, the composite covariants are equivalent to $10-5, =5$, the full number of the asyzygetic covariants. And similarly the theory as it stands gives a non-existent irreducible covariant $(a, \ldots)^8(x, y)^{20}$. The theory being thus in error, by reason that it

omits to take account of the interconnexion of the syzygies, there is no difficulty in conceiving that the effect is the introduction of an infinite series of non-existent irreducible covariants, which, when the error is corrected, will disappear, and there will be left only a finite series of irreducible covariants.

Although I am not able to make this correction in a general manner so as to show from the theory that the number of the irreducible covariants is finite, and so to present the theory in a complete form, it nevertheless appears that the theory can be made to accord with the facts; and I reproduce the theory, as well to show that this is so as to exhibit certain new formulæ which appear to me to place the theory in its true light. I remark that although I have in my second memoir considered the question of finding the number of irreducible covariants of a given degree θ in the coefficients but of any order whatever in the variables, the better course is to separate these according to their order in the variables, and so consider the question of finding the number of the irreducible covariants of a given degree θ in the coefficients, and of a given order μ in the variables. (This is, of course, what has to be done for the enumeration of the irreducible covariants of a given quantic; and what is done completely for the quadric, the cubic, and the quartic, and for the quintic up to the degree 6 in my Eighth Memoir (Phil. Trans. 1867). The new formulæ exhibit this separation; thus (Second Memoir, No. 49), writing a instead of x, we have for the quadric the expression $\dfrac{1}{(1-a)(1-a^2)}$, showing that we have irreducible covariants of the degrees 1 and 2 respectively, viz. the quadric itself and the discriminant: the new expression is $\dfrac{1}{(1-ax^2)(1-a^2)}$, showing that the covariants in question are of the actual forms $(a, \ldots \between x, y)^2$ and $(a, \ldots)^2$ respectively. Similarly for the cubic, instead of the expression No. 55, $\dfrac{1-a^6}{(1-a)(1-a^2)(1-a^3)(1-a^4)}$, we have $\dfrac{1-a^6x^6}{(1-ax^3)(1-a^2x^2)(1-a^3x^3)(1-a^4)}$, exhibiting the irreducible covariants of the forms $(a, \ldots \between x, y)^3$, $(a, \ldots)^2 (x, y)^2$, $(a \ldots)^3 (x, y)^3$, and $(a, \ldots)^4$, connected by a syzygy of the form $(a, \ldots)^6 (x, y)^6$; and the like for quantics of a higher order.

In the present Ninth Memoir I give the last-mentioned formulæ; I carry on the theory of the quintic, extending the Table No. 82 of the Eighth Memoir up to the degree 8, calculating all the syzygies, and thus establishing the interconnexions, in virtue of which it appears that there are really no irreducible covariants of the forms $(a, \ldots)^8 (x, y)^{14}$, and $(a, \ldots ^8 \between x, y)^{20}$. I reproduce in part Prof. Gordan's theory so far as it applies to the quintic; and I give the expressions of such of the 23 covariants as are not given in my former memoirs; these last were calculated

for me by Mr. W. Barrett Davis, by the aid of a grant from the Donation Fund at the disposal of the Royal Society. The paragraphs of the present memoir are numbered consecutively with those of the former memoirs on Quantics.

II. "On the Cause and Theoretic Value of the Resistance of Flexure in Beams." By W. H. BARLOW, F.R.S. Received April 13, 1870.

(Abstract.)

The author refers to his previous papers, read in 1855 and 1857, wherein he described experiments showing the existence of an element of strength in beams, which varied with the degree of flexure, and acts in addition to the resistance of tension and compression of the longitudinal fibres. It was pointed out that the ratio of the actual strength of solid rectangular beams to the strength as computed by the theory of Liebnitz is,

In cast iron, as about $2\frac{1}{4}$ to 1.
In wrought iron as $1\frac{3}{5}$ and $1\frac{3}{4}$ to 1.
And in steel, as $1\frac{3}{5}$ and $1\frac{3}{4}$ to 1.

The theory of Liebnitz assumes a beam to be composed of longitudinal fibres only, contiguous, but unconnected, and exercising no mutual lateral action. But it is remarked that a beam so constituted would possess no power to resist transverse stress, and would only have the properties of a rope.

Cast iron and steel contain no actual fibre, and wrought iron (although some qualities are fibrous) is able to resist strain nearly equally in any direction.

The idea of fibre is convenient as facilitating investigation ; but the word fibre, as applied to a homogenous elastic solid, must not be understood as meaning filaments of the material. In effect it represents lines of direction, in which the action of forces can be ascertained and measured; for in torsion-shearing and "*angular deformation*" the fibres are treated by former writers as being at the angle of 45°, because it has been shown that the diagonal resistances have their greatest manifestation at that angle.

Elastic solids being admitted to possess powers of resistance in the direction of the diagonals, attention is called to omission of the effect of resistance in the theory of beams.

The author then states, as the result of his investigation, that compression and extension of the diagonal fibres constitute an element of strength equal to that of the longitudinal fibres, and that *flexure* is the consequence of the relative extensions and compressions in the direct and diagonal fibres, arising out of the amount, position, and direction of applied forces.

Pursuing the subject, it is shown that certain normal relations subsist